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Model Fitting and Comparison for Repeated Measurement Data from Eye Tracking Experiment

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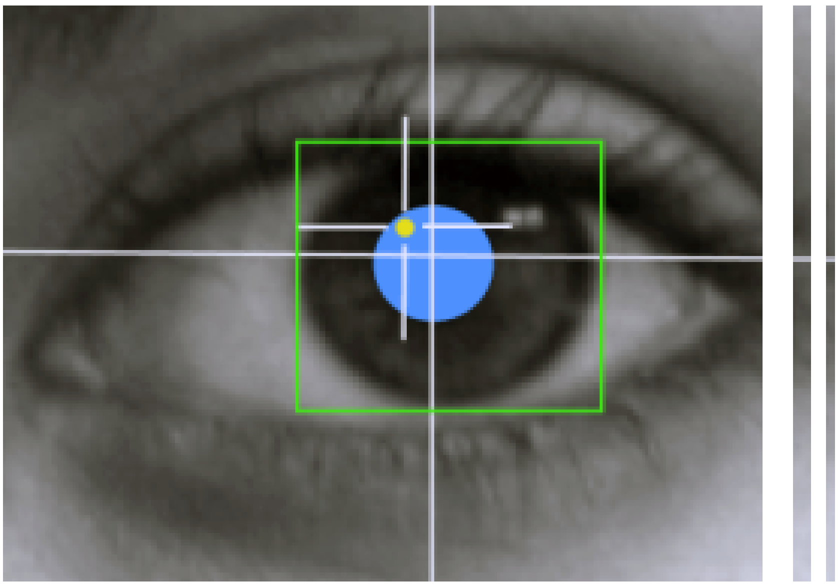


1 ABSTRACT

The data from an eye tracking experiment is introduced. Both linear models and mixed models are used to fit the data. Different covariance structures for random effects are used. The goodness of fitting is compared for different scenarios. Residual analysis for chosen models is also conducted.

2 INTRODUCTION

2.1 THE EXPERIMENT

Schizophrenia is one of the most pervasive of severe psychological diseases. Current psychological theory suggests the responses from schizophrenics suffer from a deficit, which makes them similar to the responses from non-schizophrenics but relatively slower (e.g. due to motor problems). To measure the latency of responses, psychologists Prof. Philip Holzman and Dr. Deborah Levy designed and conducted an eye-tracking experiment. In the experiment, the head of patient is fixed and use his or her eyes to track a visual target that moves back and forth along a horizontal line on a screen in front of him or her. The outcome measurement is called gain ration, which is eye velocity divided by target velocity, and it is recorded repeatedly at the peak velocity of the target. Also, there are three types of different conditions under which the experiment is conducted. The first type is PS (plain sine), which means the target velocity is proportional to the sine of time (figure 2.1) and the color of the target is plain. The second condition is CS (color sine), which means the target moves in the same as in PS but the colors of target keep changing from white to orange or blue. The third condition is TR (triangular) in which the target moves at a constant speed equal to the peak speed of PS (figure 2.2) and the color of target is always white.

2.2 THE DATA SET

The data set collected from the experiment has 60 observations each of which consist of the results from a 11-cycle-per-trial eye tracking experiment. For three types of conditions, these observations are distributed as: PS=34, CS=17, TR=9. And there are 31 patients (22 female and 9 male) in the experiments.

Figure 2.1: Target movement for PS and CS

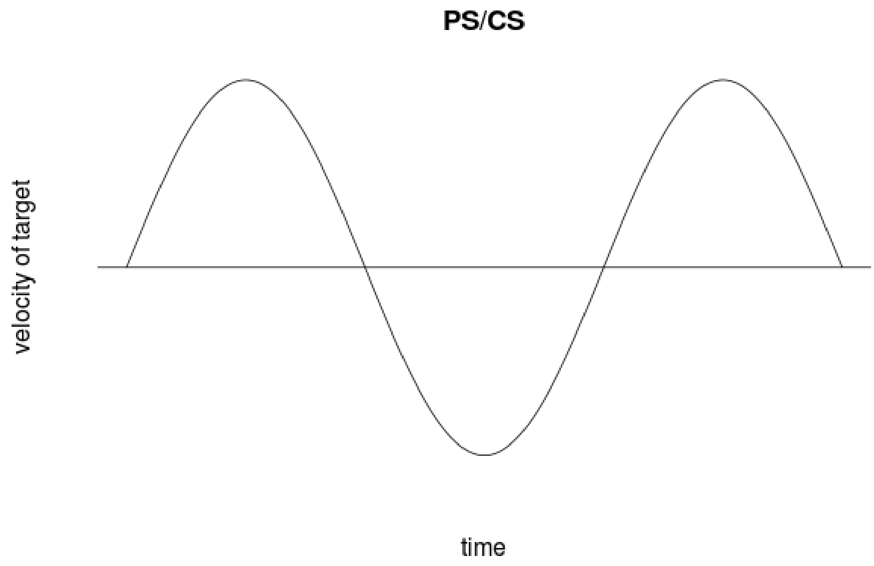


Figure 2.2: Target movement for TR

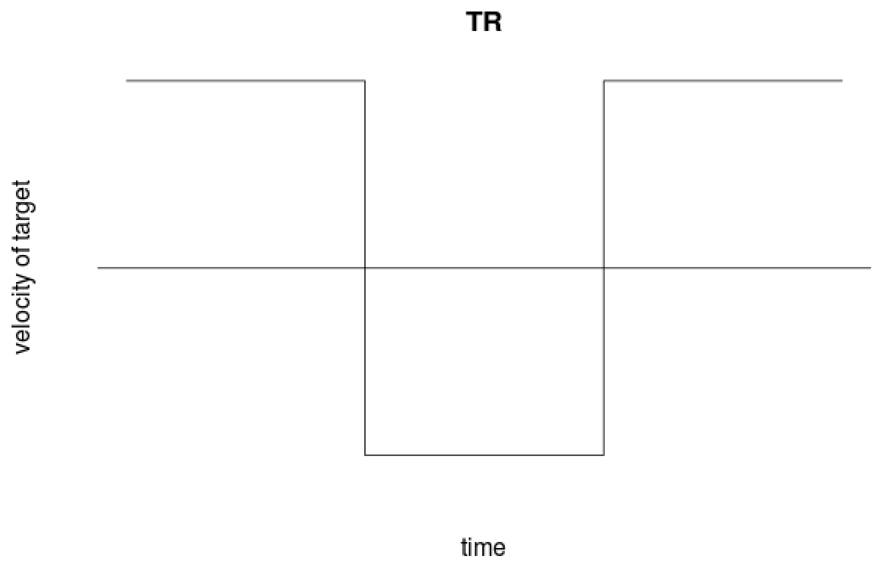
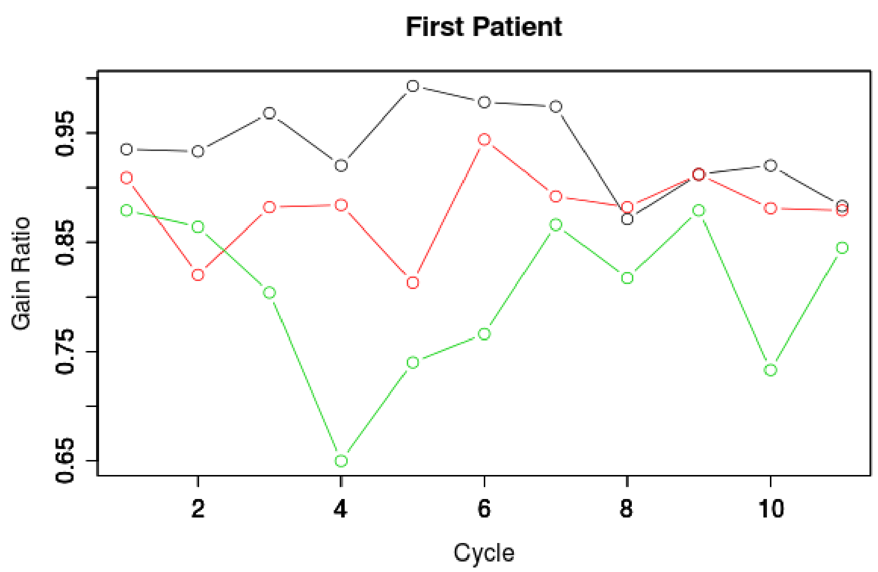


Table 2.1: Sample data for 3 trials [sex: 0=male, 1=female]

ID	Sex	Type	Cycle 1	Cycle 2	...	Cycle 11
7	1	PS	0.935	0.933	...	0.883
12	0	PS	0.952	1.040	...	0.992
12	0	CS	1.030	1.010	...	1.010

Sample data for 3 trials is showed in table 2.1. The result for the first patient is shown in figure 2.3 and the result for all the patients regarding to sex and condition type is shown in figure 2.4.

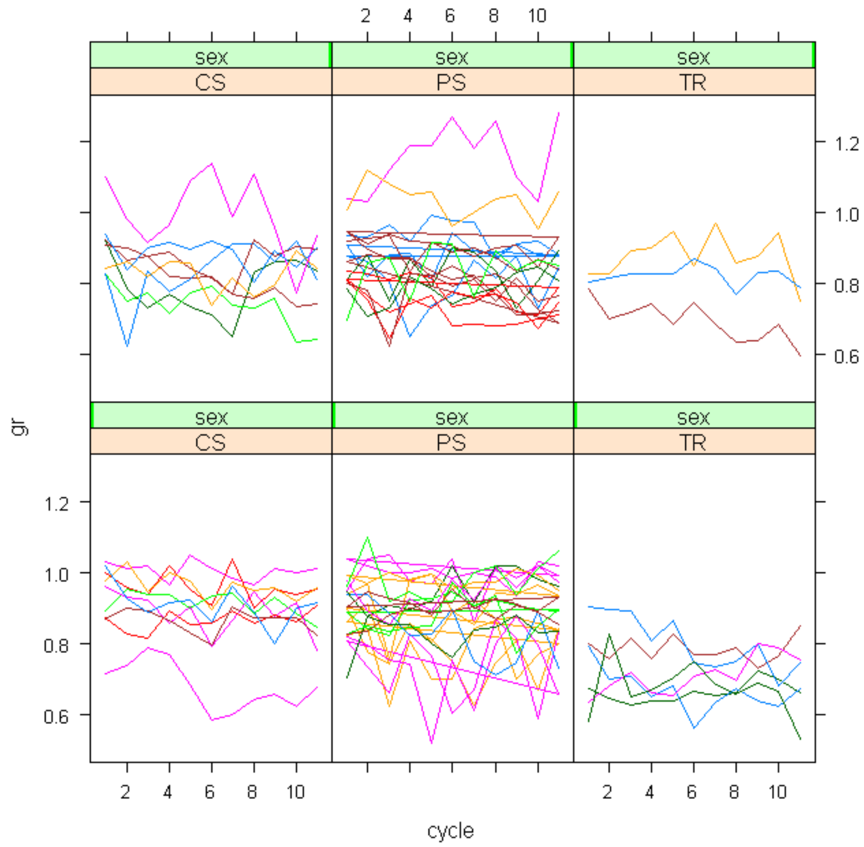
Figure 2.3: The results for the first patient

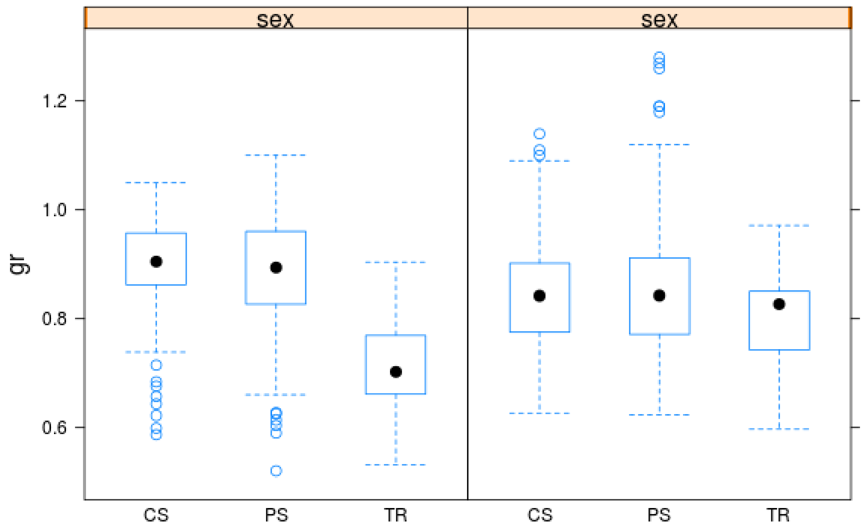


3 Methodology & Results

The data introduced in previous section is a typical repeated measurement data in which each patient is tested for multiple trials under different con-

Figure 2.4: The results for the all patients (categorized by "Sex" and "Type")





ditions and for each trial the patient is tested for 11 cycles. The interest of model building is to find out how the response time varies regarding to individual patient, the gender, the experimental condition, and the repeated cycles. The final goal of this study is to test a variety of models and identify the best model that explains the data well.

3.1 Models

Both linear and mixed models are proposed in the first place. The general forms for these models are shown in below.

Linear models

$$Y = X\beta + \epsilon \tag{3.1}$$

$$y_i = \beta_0 + \beta_1 x_{i1} + \ldots + \beta_1 x_{ip} + \epsilon_i \tag{3.2}$$

Mixed models

$$Y_i = X_i\beta + Z_i b_i + \epsilon \tag{3.3}$$

$$y_{ij} = \beta_0 + \beta_1 x_{i1} + \ldots + \beta_1 x_{ip} + b_{1i} z_{1i} + \ldots + b_{ki} z_{ki} + \epsilon_{ij} \tag{3.4}$$

where

- i indicates the i^{th} patient;
- j indicates the j^{th} repeated measurement;
- $X\beta$ indicates the fixed effect;
- Zb indicates the random effect;
- ϵ indicates error term.

and $b_i \sim N(0, D)$, $\epsilon_i \sim N(0, \Sigma)$. b_i and ϵ_i are statistically independent and their covariance matrix could have different structures.

Table 3.1: Linear model fitting [s=sex, t=type, c=cycle]

Model	Linear terms	Interaction	Log-likelihood	AIC
1	s+t	null	546.806	-1083.611
2	s+t	s*t	558.759	-1103.518
3	s+t+c	s*t	560.743	-1105.486
4	s+t+c	s*c	549.368	-1082.737
5	s+t+c	s*c	549.024	-1084.048
6	s+t+c	s*t+s*c+t*c	562.140	-1098.280

3.2 AIC and Log-likelihood

AkaikeâĂŹs Information Criterion (AIC), developed by Hirotsugu Akaike, is a measure of relative goodness of fit of an estimated statistical model. The AIC is defined as

$$AIC = -2\log(L) + 2k \tag{3.5}$$

where k is the number of parameters in the model and L is the maximized value of the likelihood function for the estimated model. For model selection, AIC rewards the goodness of fit and in the meantime includes a penalty for model complexity (the number of parameters). This penalty reduces the danger of overfitting. In general case, lower AIC stands for better fit.

3.3 Results

3.3.1 Linear models

First, six linear models in table 3.1 are fitted. By comparing Log-likelihood and AIC, two models (model 3 and 6) have better performance than others. However, in these models some estimated coefficients are not significant ($p-value > 0.05$). And from further analysis we will see linear models indeed cannot fit this data set well.

3.3.2 Mixed models

Then, six mixed effect models in table 3.2 are fitted using maximum likelihood estimation. And from the fitting results, we found that "sex" and "typePS", especially "sex", are not significant ($p-value > 0.05$).

Table 3.2: Mixed models fitting [i=intercept, PS=typePS]

Model	Fixed effect	Random effect	Log-likelihood	AIC	$p-value > 0.05$
1	s+t	i	824.260	-1636.520	s
2	s+t	c	743.860	-1457.721	s
3	s+t	i+c	842.505	-1669.010	s
4	s+t+s*t	i	828.327	-1640.654	s, s*PS
5	s+t+s*t	c	751.699	-1487.398	PS, s*PS
6	s+t+s*t	i+c	847.052	-1674.104	s

Table 3.3: Mixed models without "sex" [t=type, c=cycle, i=intercept]

Model	Fixed effect	Random effect	Log-likelihood	AIC	$p-value > 0.05$
1	t	i	823.699	-1637.397	null
2	t	c	743.560	-1477.120	null
3	t	i+c	841.966	-1669.933	null

So, "sex" is removed from the model and another three mixed effect models without "sex" are fitted. From the result in table 3.3, we can see that none of these three models contains insignificant term and both Log-likelihood and AIC suggest Model 3 is the best model.

Furthermore, the best mixed effect model is reconsidered by fitting model with four different covariance structures. The result of Log-likelihood and AIC, shown in table 3.4, suggests AR(1) is the covariance structure that fits the data best.

Thus, the best model at this stage is the mixed model with AR(1) covariance structure, shown in (3.6).

Table 3.4: Mixed effect models with different covariance structures

Covariance Structure	Log-likelihood	AIC
Compound Symmetric	841.966	-1667.933
AR(1)	867.633	-1719.267
Linear Spatial	861.982	-1707.924
Gaussian Spatial	862.187	-1708.374

Table 3.5: Estimation of the fixed effect terms

	Est. value	Std. error	DF	T-value	p-value
Intercept	0.848	0.017	627	50.333	0e+00
typePS	0.033	0.009	627	3.684	2e-04
typeTR	-0.115	0.013	627	-8.744	0e+00

Table 3.6: Comparison of prediction efficiency

Model	LM3	LM6	MM+AR(1)
MSE	0.0789	0.0787	0.0439
MAE	0.0107	0.0107	0.0034

$$y_{ij} = 0.848 + 0.033 \times \text{typePS} - 0.115 \times \text{typeTR} + b_{i0} + b_{i1} \times \text{cycle} + \epsilon_{ij} \quad (3.6)$$

where

$$\begin{pmatrix} b_{i0} \\ b_{i1} \end{pmatrix} \sim N(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.00755 & -0.23300 \\ -0.23300 & 0.00003 \end{pmatrix})$$

and the parameter for AR(1) structure is $\hat{p} = 0.292$. The details of fixed effect terms are shown in table 3.5.

3.3.3 Prediction error and residual analysis

Both mean of squared errors of prediction (MSE) and mean of absolute errors of prediction (MAE) are used to compare linear model 3 and 6 (in table 3.1) and the best mixed model. The result is shown in table 3.6, which clearly suggests that the best mixed model has much better prediction efficiency.

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \quad (3.7)$$

$$MAE = \frac{1}{n} \sum_{i=1}^{n} |y_i - \hat{y}_i| \quad (3.8)$$

The scatter-plots (figure 3.1) between true Y and predicted Y for LM 6 and the best mixed model reveal the result more clearly: the scatter dots for the best mixed model spread much closer around the perfect-predict-line (red line) than LM 6. And the residual-plots with 3σ bound (figure 3.2) confirm the conclusion : residual dots for the best mixed model spread much closer around zero-residual-line (middle red line) than LM 6; and 3σ bounds (green lines) for best mixed model is much narrower than LM 6.

4 Conclusion

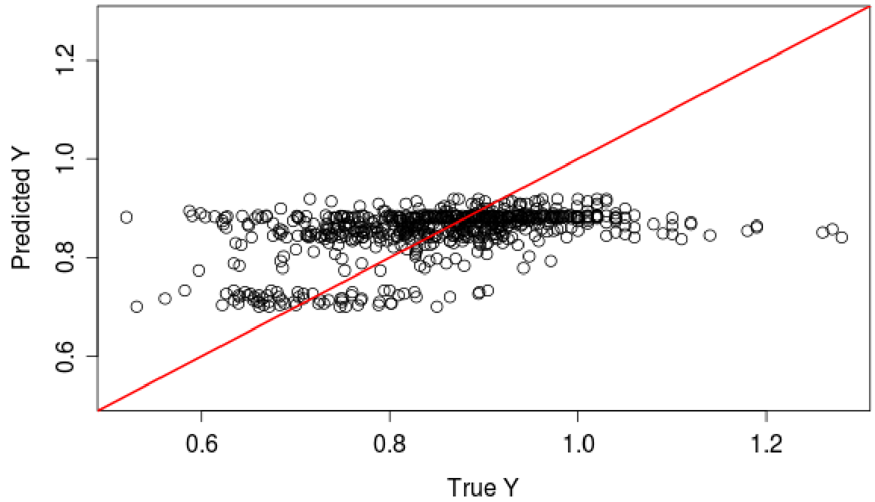
From the results of model fitting and comparison, we can conclude that the mixed model fits data much better than linear model for two possible reasons: first, there exists random effects that come from repeated measurements; second, each trial of the experiment consists of 11 continuous cycles and AR(1) covariance structure can capture correlations between these cycles. Also, "sex" doesn't have significant influence on gain ratio measurement. However, gain ratio measurement does vary for different types of conditions under which trials are conducted.

5 Appendix

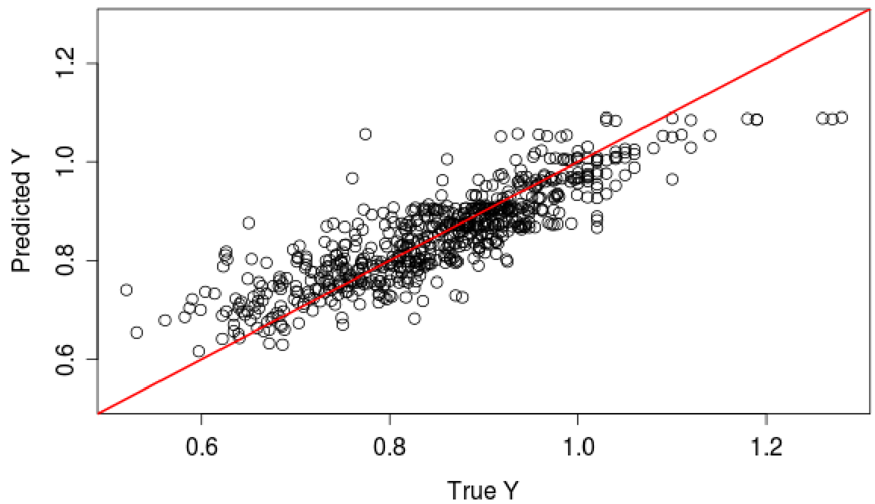
Some R output (the complete source code is attached in separate file)

```
> library(nlme)

> gr.ar1 <- lme(gr~type,data=eye.dat,
        random=~(cycle)|id,method="ML",corr=corAR1())

> summary(gr.ar1)
Linear mixed-effects model fit by maximum likelihood
 Data: eye.dat
        AIC       BIC     logLik
  -1719.267 -1683.329   867.6333

Random effects:
 Formula: ~(cycle) | id
 Structure: General positive-definite, Log-Cholesky parametrization
                StdDev     Corr
```

Figure 3.1: True vs. Predicted Y

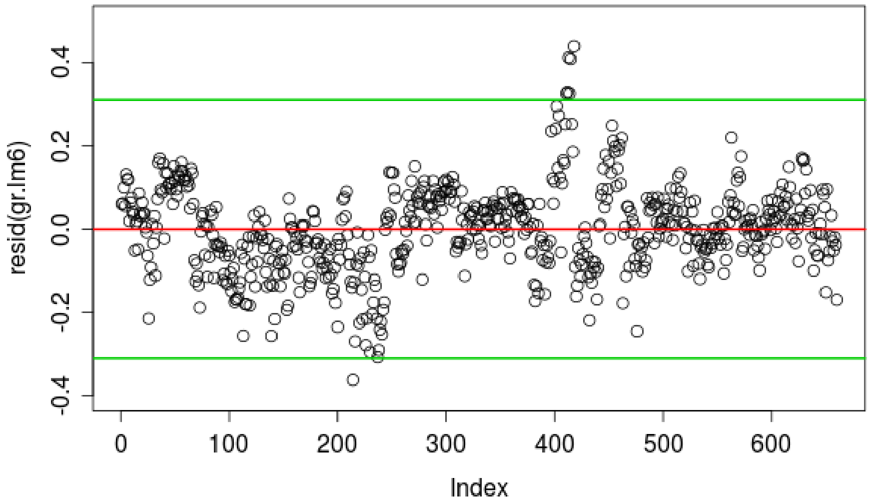


LM 6

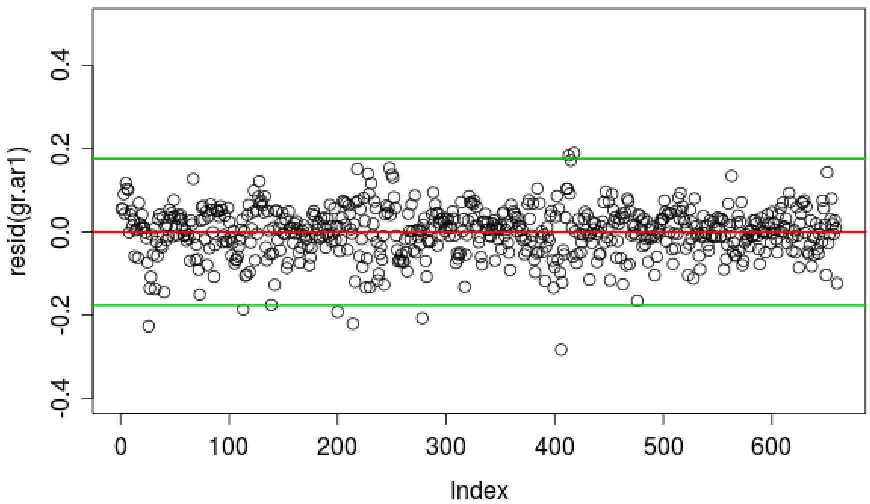


The best mixed model

Figure 3.2: Residual plots



LM 6



The best mixed model

```
(Intercept) 0.086896731   (Intr)
cycle       0.005786277   -0.233
Residual    0.062234969

Correlation Structure: AR(1)
 Formula: ~1 | id
 Parameter estimate(s):
     Phi
0.307535
Fixed effects: gr ~ type
              Value      Std.Error      DF    t-value  p-value
(Intercept)  0.8480172    0.016848298   627   50.33251   0e+00
typePS       0.0332751    0.009032632   627   3.68388    2e-04
typeTR      -0.1145947    0.013105869   627  -8.74377    0e+00
 Correlation:
               (Intr)   typePS
typePS  -0.338
typeTR  -0.249  0.429

Standardized Within-Group Residuals:
Min           Q1           Med          Q3           Max
-4.54000150  -0.49944216  0.06486762  0.58586233  3.04486906

Number of Observations: 660
Number of Groups: 31
```