Sample Size
for measuring process time

for Dr. Don Gomez

by Statistical Consulting Center

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The Engineering Problem

- The processing time of an assembly task varies.
- The goal is to determine whether the mean is less than 30 seconds, with three requirements:
  - 0.05 significance level
  - at least 90% chance of declaring the mean to be less than 30 if the true mean ≤ 29.5
  - confidence interval ≤ 0.8

Factors affecting the sample size

- In statistical language, the problem can be set up as

$$H_0: \ \mu \geq 30 \ \text{ vs } \ H_a: \mu < 30$$

- Factors affecting the sample size
  - Significant level
  - Power
  - Confidence interval
  - variance

Population Variance on Sample Size

- Population variance describes individual variation in population, less data variation means less sample size we need if we would like to obtain the same accuracy and power.

- Ways to obtain population variance:
  - by past experiments
  - using its substitute ----sample variance
  - a reasonable guess

Estimation of Variance

- A reasonable guess of population standard deviation for approximate normal distribution is:

$$\sigma = \frac{\text{range}}{6} = \frac{\max imum - \min imum}{6}$$

- In our case the guessed standard deviation is:

$$\sigma_{guess} = \frac{9}{6} = 1.5$$

Methodology: *Under Power Constraint*

- At significance level $\alpha = 0.05$, reject H_0 if $T = \dfrac{\overline{X} - 30}{S/\sqrt{n}} \le -t_{n-1,0.05}$
- The power function is

$$\beta(\mu, n) = P\left(T_{n-1,\delta} \le -t_{n-1,0.05}\right)$$

  where $\delta = \dfrac{\mu - 30}{\sigma/\sqrt{n}}$ is the noncentrality parameter.
- The power constraint $\beta(29.5, n) \ge 0.90$
- Replacing t distribution by normal distribution, one can find an initial guess of n by $n \ge (5.854\sigma)^2$
- Increase n until $\beta(29.5, n) \ge 0.90$ is satisfied.

Methodology: *Under CI Constraint*

- The 95% confidence interval is

$$\bar{X} \pm t_{n-1,0.025} S / \sqrt{n}$$

- The expected width is

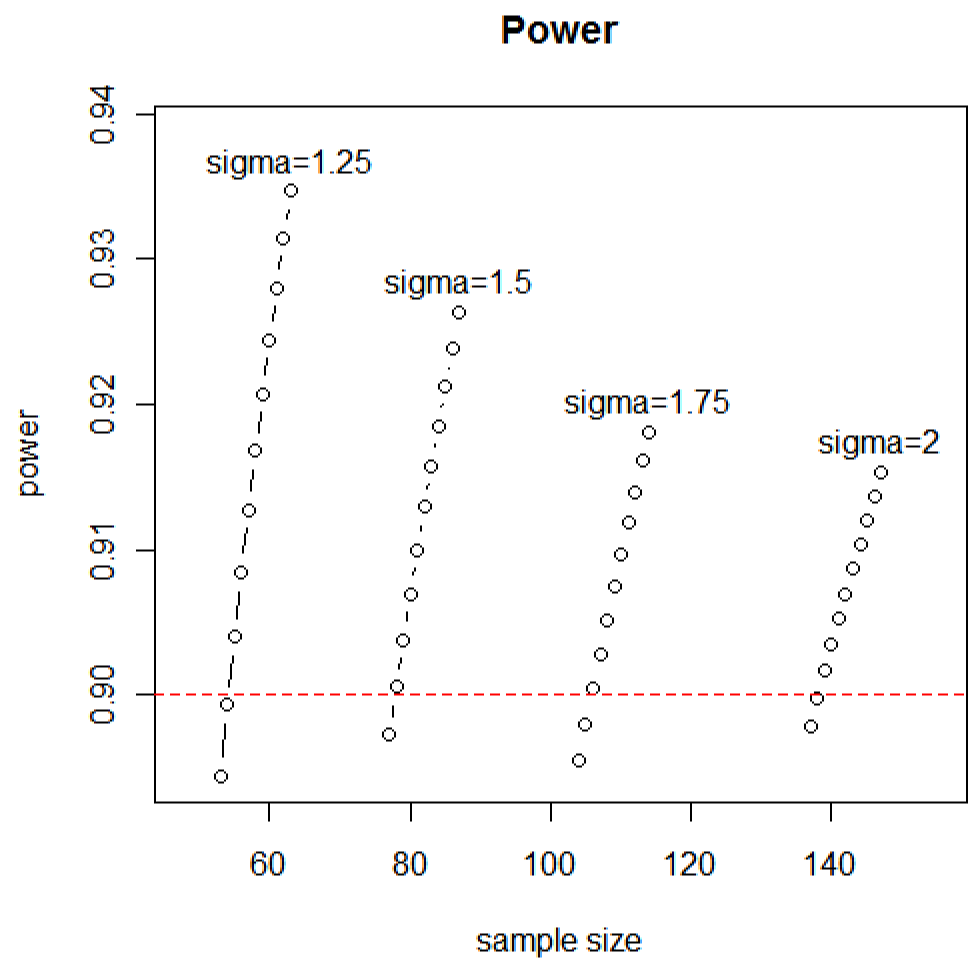
$$L(n) = 2t_{n-1,0.025}\, E(S) \big/ \sqrt{n}$$

where $E(S) = \dfrac{\sqrt{2}\,\Gamma(n/2)\,\sigma}{\sqrt{n-1}\,\Gamma\left[(n-1)/2\right]}$

- The CI constraint $L(n) \le 0.8$
- Based on the preliminary guess of σ, one can find an initial guess of n by $n \ge (4.90\sigma)^2$
- Increase n until $L(n) \le 0.8$ is satisfied.

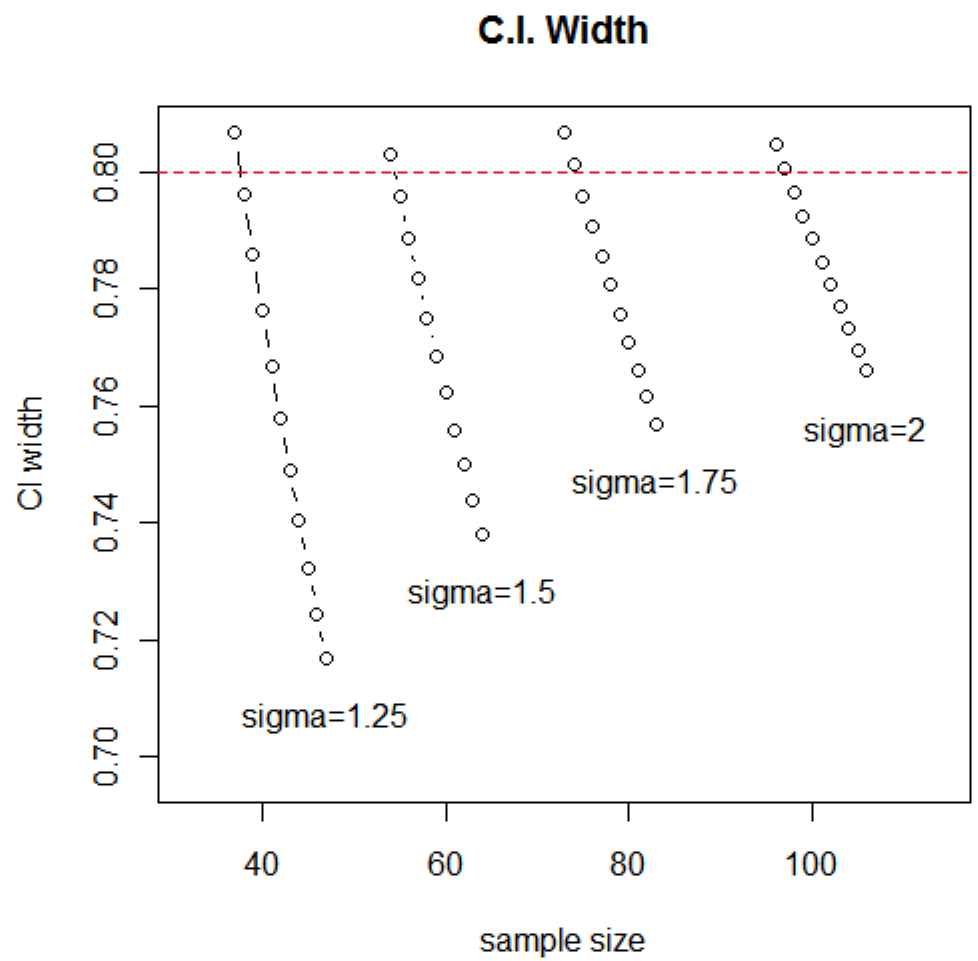
For POWER>0.90

Larger sample size is required when sigma is larger!

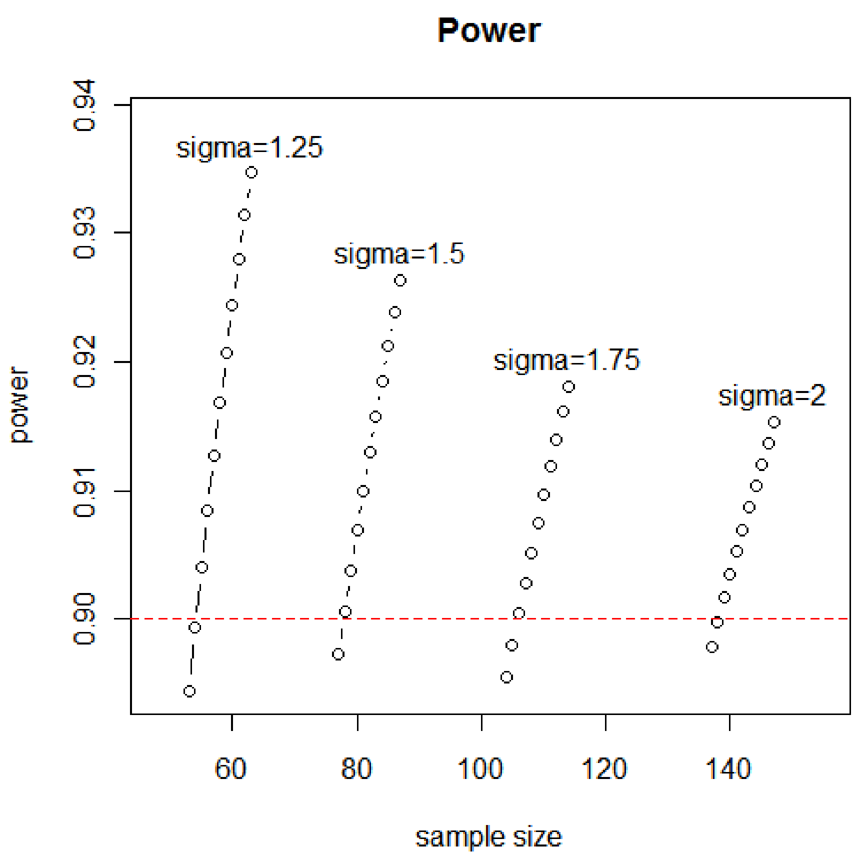


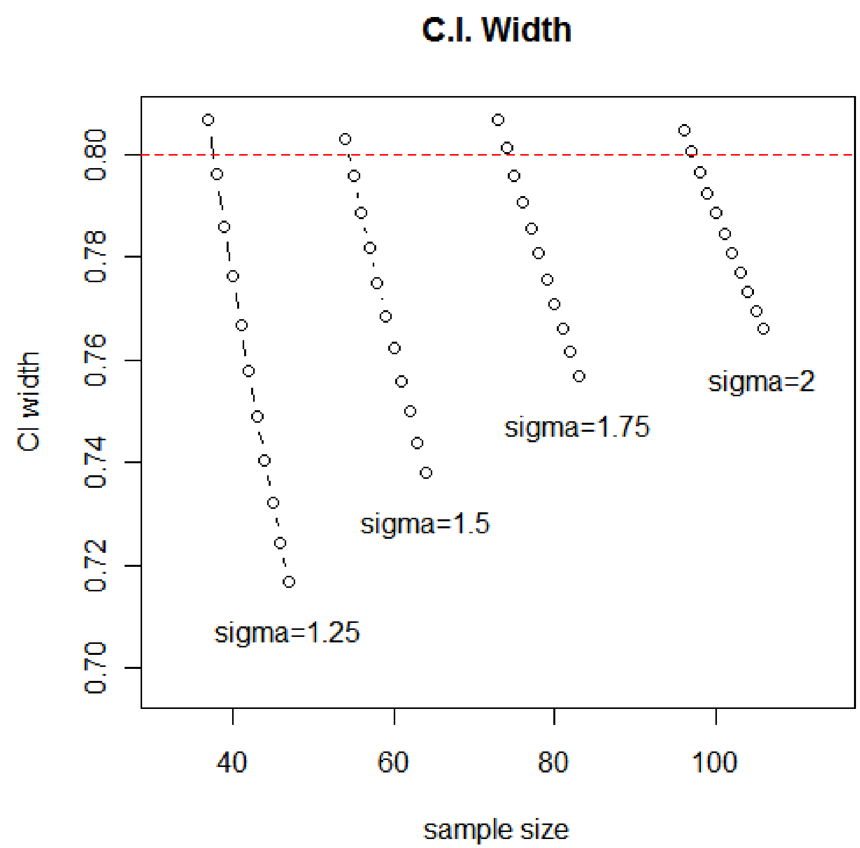
For WIDTH<0.80

Larger sample size is required when sigma is larger!



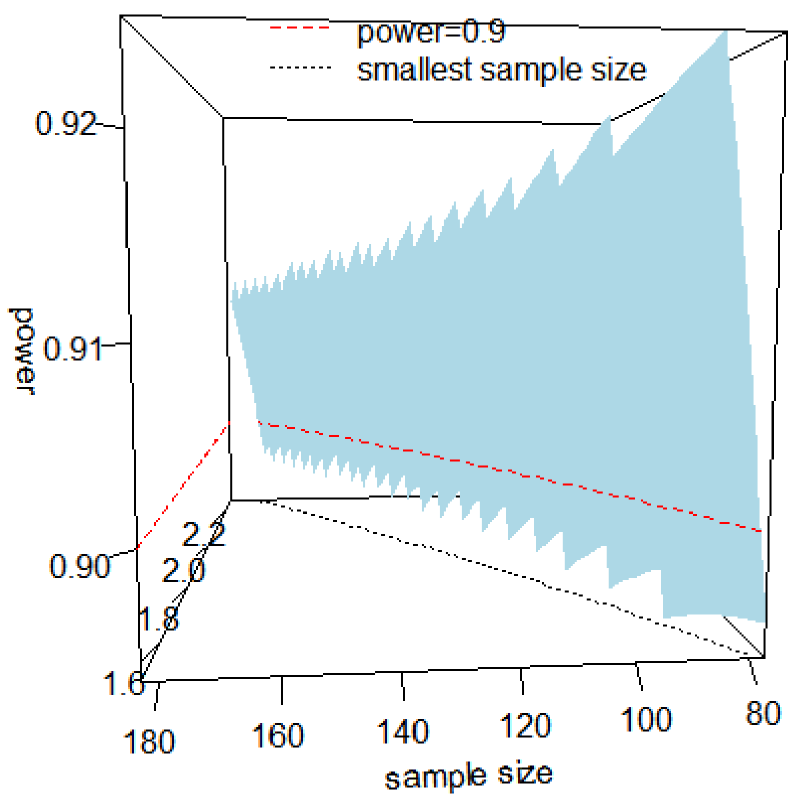
Comparison

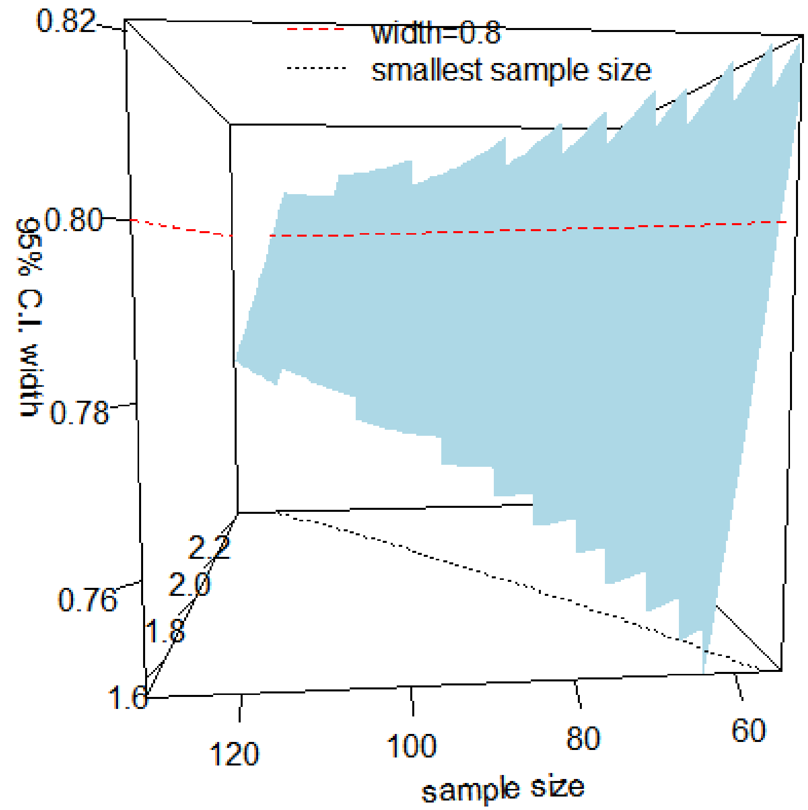




For the same sigma, larger sample size is required to achieve POWER>0.9 than to achieve WIDTH<0.8!

More details in 3-D





Summary: Under Power Constraint

By (3.6)

$$n \geq (5.854\hat{\sigma})^2 = (5.854 \times 1.5)^2 = 77.1$$

So, we search for the smallest n to satisfy

$$P(T_{n-1,\delta} \leq -t_{n-1,0.05}) \geq 0.90$$

by starting with initial n = 78

SAS Output (Under Power Constraint)

n	df	delta	cp	power
78	77	-2.94392	-1.66488	0.89850

n	df	delta	cp	power
79	78	-2.96273	-1.66462	0.90183

Summary: Under CI Constraint

By (3.11)

$$n \geq (4.90\sigma)^2 = (4.90 \times 1.5)^2 = 54.0$$

So, we search for the smallest n to satisfy

$$2t_{n-1,0.025} \frac{2^{1/2}\Gamma(n/2)\sigma}{n^{1/2}(n-1)^{1/2}\Gamma\left[(n-1)/2\right]} \leq 0.8$$

by starting with initial n = 54

SAS Output (Under CI Constraint)

n	df	cp	width
54	53	2.00575	0.81499

n	df	cp	width
55	54	2.00488	0.80727

n	df	cp	width
56	55	2.00404	0.79976

Finalizing the Sample Size

Since we have to satisfy both constraints, we adapt the maximum of the sample sizes just found.

Therefore, the sample size of **79** or more is recommended to Mr. Gomez.

SAS Codes

```
data one;
n=78;
df=n-1;
delta=-0.5/1.5*sqrt(n);
cp=tinv(0.05,df);
power=probt(cp, df, delta);
put cp power;
proc print; run;

data two;
n=54;
df=n-1;
cp=-tinv(0.025, df);
width=2*cp*sqrt(2/(n*(n-1)))*gamma(n/2)*1.5/gamma((n-1)/2);
put cp width;
proc print; run;
```

- Thank You!

**see attached files for more details.*